Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Wednesday 9 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35384A

1. $$y = 3^x + 2x.$$

(*a*) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\ \mathrm{d}x$.

(4)

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2. $$\mathrm{f}(x) = 3x^3 - 5x^2 - 58x + 40.$$

(*a*) Find the remainder when $\mathrm{f}(x)$ is divided by $(x - 3)$.

(2)

Given that $(x - 5)$ is a factor of $\mathrm{f}(x)$,

(*b*) find all the solutions of $\mathrm{f}(x) = 0$.

(5)

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3. $$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

(*b*) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

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4. (*a*) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1+ ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(*b*) find the possible values of a.

(2)

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5. (*a*) Given that $5 \sin \theta = 2 \cos \theta$, find the value of $\tan \theta$.

(1)

(*b*) Solve, for $0 \leq x < 360°$,

$$5 \sin 2x = 2 \cos 2x \text{ ,}$$

giving your answers to 1 decimal place.

(5)

6.

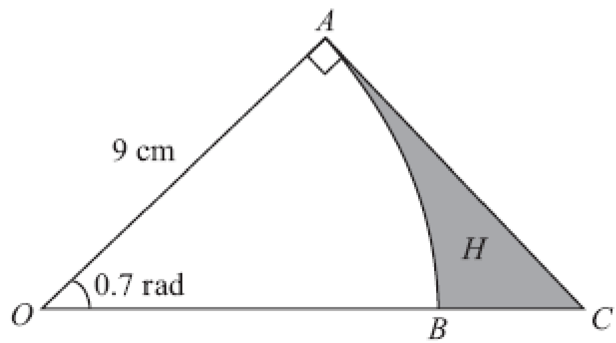

Figure 1

Figure 1 shows the sector *OAB* of a circle with centre *O*, radius 9 cm and angle 0.7 radians.

(*a*) Find the length of the arc *AB*.

(2)

(*b*) Find the area of the sector *OAB*.

(2)

The line *AC* shown in Figure 1 is perpendicular to *OA*, and *OBC* is a straight line.

(*c*) Find the length of *AC*, giving your answer to 2 decimal places.

(2)

The region *H* is bounded by the arc *AB* and the lines *AC* and *CB*.

(*d*) Find the area of *H*, giving your answer to 2 decimal places.

(3)

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7. (*a*) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(*b*) Hence, or otherwise, solve $2\log_3(x-5) - \log_3(2x-13) = 1$.

(2)

8.

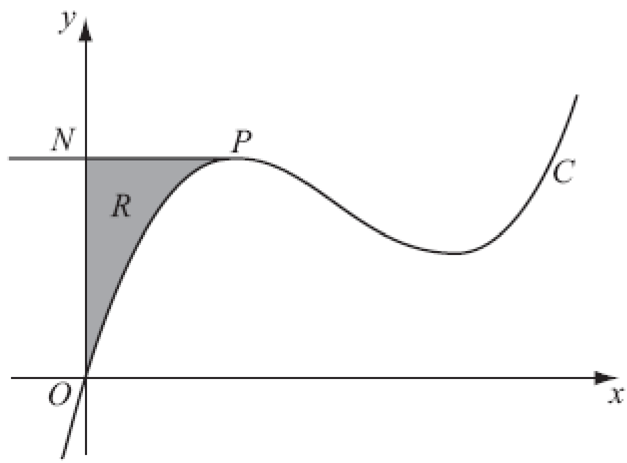

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(*a*) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(*b*) Use calculus to find the exact area of R.

(6)

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9. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(*a*) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(*b*) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(*c*) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(*d*) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(*e*) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)

10. The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$.

(a) Find an equation for C.

(4)

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1 .

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB.

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	(a) 2.35, 3.13, 4.01 (One or two correct B1 B0, all correct B1 B1) Important: If part (a) is blank, or if answers have been crossed out and no replacement answers are visible, please send to Review as 'out of clip'.	B1 B1 (2)
	(b) $\frac{1}{2}\times 0.2$ ...... (or equivalent numerical value)	B1
	$k\{(1+5)+2(1.65+p+q+r)\}$, k constant, $k\neq 0$ (See notes below)	M1 A1
	$= 2.828$ (awrt 2.83, allowed even after minor slips in values)	A1
	The fractional answer $\frac{707}{250}$ (or other fraction wrt 2.83) is also acceptable. Answers with no working score no marks.	(4) **6**
2	(a) Attempting to find $\mathrm{f}(3)$ or $\mathrm{f}(-3)$	M1
	$\mathrm{f}(3)=3(3)^3-5(3)^2-(58\times 3)+40=81-45-174+40=-98$	A1 (2)
	(b) $\{3x^3-5x^2-58x+40=(x-5)\}$ $(3x^2+10x-8)$	M1 A1
	Attempt to factorise 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic.	M1
	$(3x-2)(x+4)=0$ $x=....$ or $x=\frac{-10\pm\sqrt{100+96}}{6}$	A1 ft
	$\frac{2}{3}$ (or exact equiv.), -4, 5 (Allow 'implicit' solns, e.g. f(5) = 0, etc.)	A1 (5)
	Completely correct solutions without working: full marks.	**7**
3	(a) $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$ $2x-\frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. $+C$, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$)	M1
	$8-\frac{k}{4}<0$ $k>32$ (or $32<k$) Correct inequality needed	A1
		(2) **4**

Question Number	Scheme	Marks
4	(a) $(1+ax)^7 = 1+7ax....$ or $1+7(ax)....$ (<u>Not</u> unsimplified versions)	B1
	$+\frac{7\times6}{2}(ax)^2+\frac{7\times6\times5}{6}(ax)^3$ Evidence from <u>one</u> of these terms is enough	M1
	$+21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$	A1
	$+35a^3x^3$ or $+35(ax)^3$ or $+35(a^3x^3)$	A1
		(4)
	(b) $21a^2 = 525$	M1
	$a = \pm5$ (Both values are required)	A1
	(The answer a = 5 with no working scores M1 A0)	(2)
		6
5	(a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found)	B1 (1)
	Requires the correct value with no incorrect working seen.	
	(b) awrt 21.8 (α)	B1
	(Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft)	
	(This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)	
	$180+\alpha$ $(=201.8)$, or $90+(\alpha/2)$ (if division by 2 has already occurred)	M1
	(α found from $\tan 2x = ...$ or $\tan x = ...$ or $\sin 2x = \pm...$ or $\cos 2x = \pm...$)	
	$360+\alpha$ $(=381.8)$, or $180+(\alpha/2)$	M1
	(α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$)	
	OR $540+\alpha$ $(=561.8)$, or $270+(\alpha/2)$	
	(α found from $\tan 2x = ...$)	
	Dividing at least one of the angles by 2	M1
	(α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$)	
	$x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt)	A1 (5)
		6

Question Number	Scheme	Marks
6	(a) $r\theta = 9 \times 0.7 = 6.3$ (Also allow 6.30, or awrt 6.30)	M1 A1 (2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 81 \times 0.7 = 28.35$ (Also allow 28.3 or 28.4, or awrt 28.3 or 28.4) (Condone 28.35^2 written instead of $28.35\,\text{cm}^2$)	M1 A1 (2)
	(c) $\tan 0.7 = \frac{AC}{9}$ $AC = 7.58$ (Allow awrt) NOT 7.59 (see below)	M1 A1 (2)
	(d) Area of triangle AOC = $\frac{1}{2}(9 \times \text{their } AC)$ (or other complete method) Area of R = "34.11" – "28.35" (triangle – sector) or (sector – triangle) (needs a value for each) = 5.76 (Allow awrt)	M1 M1 A1 (3) **9**
7	(a) $2\log_3(x-5) = \log_3(x-5)^2$ $\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \frac{(x-5)^2}{2x-13}$ $\log_3 3 = 1$ seen or used correctly $\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q$ $\left\{\frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)\right\}$ $x^2 - 16x + 64 = 0$ (*)	B1 M1 B1 M1 A1 cso (5)
	(b) $(x-8)(x-8) = 0 \Rightarrow x = 8$ Must be seen in part (b). Or: Substitute $x = 8$ into original equation and verify. Having additional solution(s) such as $x = -8$ loses the A mark. $x = 8$ with no working scores both marks.	M1 A1 (2) **7**

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>8</td><td>(a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)
At x = 2, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ k = 28 (*)
N.B. The '= 0' must be seen at some stage to score the final mark.

Alternatively: (using k = 28)
$\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)
'Assuming' k = 28 only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at x = 2 represents the maximum turning point.</td><td>M1 A1

A1 cso

(3)</td></tr>
<tr><td></td><td>(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$
$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = \ldots$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$
(With limits 0 to 2, substitute the limit 2 into a 'changed function')

y-coordinate of $P = 8 - 40 + 56 = 24$ Allow if seen in part (a)
(The B1 for 24 may be scored by implication from later working)
Area of rectangle = $2 \times$ (their y-coordinate of P)
Area of R = (their 48) $- \left(\text{their } \frac{100}{3}\right)$ $= \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$
If the subtraction is the 'wrong way round', the final A mark is lost.</td><td>M1 A1

M1

B1

M1 A1

(6)
9</td></tr>
</table>

Question Number	Scheme	Marks	
9	(a) $25\,000 \times 1.03 = 25750$ $\left\{25000 + 750 = 25750, \text{ or } 25000\dfrac{(1-0.03^2)}{1-0.03} = 25750\right\}$ (*)	B1	(1)
	(b) $r = 1.03$ Allow $\dfrac{103}{100}$ or $1\dfrac{3}{100}$ but no other alternatives	B1	(1)
	(c) $25000r^{N-1} > 40000$ (Either letter r or their r value) Allow ' = ' or '<' $r^M > 1.6 \Rightarrow \log r^M > \log 1.6$ Allow ' = ' or '<' (<u>See below</u>) OR (by change of base), $\log_{1.03} 1.6 < M \Rightarrow \dfrac{\log 1.6}{\log 1.03} < M$ $(N-1)\log 1.03 > \log 1.6$ (Correct bracketing required) (*) Accept work for part (c) seen in part (d)	M1 M1 A1 cso	 (3)
	(d) Attempt to evaluate $\dfrac{\log 1.6}{\log 1.03} + 1$ {or $25000(1.03)^{15}$ <u>and</u> $25000(1.03)^{16}$} $N = 17$ (<u>not</u> 16.9 and not e.g. $N \geq 17$) Allow '17th year' Accept work for part (d) seen in part (c)	M1 A1	 (2)
	(e) Using formula $\dfrac{a(1-r^n)}{1-r}$ with values of a and r, and n = 9, 10 or 11 $\dfrac{25000(1-1.03^{10})}{1-1.03}$ 287 000 (<u>must</u> be rounded to the nearest 1 000) Allow 287000.00	M1 A1 A1	 (3) **10**

Question Number	Scheme	Marks
10	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$	M1 A1
	$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive value)	M1
	$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100)	A1
	(Answer only scores full marks)	(4)
	(b) (Gradient of radius =) $\frac{7-1}{10-2}=\frac{6}{8}$ (or equiv.) Must be seen in part (b)	B1
	Gradient of tangent $=\frac{-4}{3}$ (Using perpendicular gradient method)	M1
	$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)	M1
	$y-7=\frac{-4}{3}(x-10)$ or equiv (ft gradient of radius, dep. on both M marks)	A1ft
	$\{3y=-4x+61\}$ (N.B. The A1 is only available as ft after B0)	
	The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here. The equation must at some stage be exact, not, e.g. $y=-1.3x+20.3$	(4)
	(c) $\sqrt{r^2-\left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.	M1
	$=\sqrt{10^2-5^2}$ or numerically exact equivalent	A1
	$PQ\ \left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	A1 (3)
		11